Lepton flavor violation in a class of lopsided SO(10) models

Ernest Jankowski* and David W. Maybury†
Department of Physics, University of Alberta, Edmonton AB T6G 2J1, Canada

(Received 27 January 2004; revised manuscript received 21 May 2004; published 17 August 2004)

A class of predictive SO(10) grand unified theories with highly asymmetric mass matrices, known as lopsided textures, which was developed to accommodate the observed mixing in the neutrino sector, can effectively determine the rate for charged lepton flavor violation (LFV), and in particular the branching ratio for $\mu \to e\gamma$. Assuming that the supersymmetric GUT breaks directly to the constrained minimal supersymmetric standard model (CMSSM), we find that in light of the combined constraints on the CMSSM parameters from direct searches and from the WMAP satellite observations, the resulting predicted rate for $\mu \to e\gamma$ in this model class can be within the current experimental bounds for low $\tan\beta$, but that the next generation of $\mu \to e\gamma$ experiments would effectively rule out this model class if LFV is not detected.

DOI: 10.1103/PhysRevD.70.035004 PACS number(s): 11.30.Pb, 12.10.−g, 12.60.Jv, 13.35.Bv

I. INTRODUCTION

Neutrinos have been observed to oscillate between flavor states [1–8], which implies neutrino mass and mixing. In addition, the combined observations suggest that both the atmospheric and solar mixing angles are nearly maximal, known as the large angle mixing solution (LMA). Interestingly, the LMA solution implies that the lepton mixing scenario is radically different from the quark sector. Specifically, $|U_{\mu 3}|$ of the Maki-Nakagawa-Sakata (MNS) matrix is much larger than $|V_{cb}|$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Over the last few years a number of models that employ the seesaw mechanism [9] in conjunction with various flavor symmetries have been developed to address this difference [10–21]. Recently, a particularly interesting and highly successful class of supersymmetric SO(10) grand unified theories (GUTs) has emerged that makes use of asymmetric mass matrices known as lopsided textures [11–13]. In these models, the charged lepton sector is responsible for the large atmospheric mixing angle while the Majorana singlet neutrino matrix has a simple form that results in the large solar mixing angle. Throughout this paper we will refer to these models as the AB model class [11].

After GUT breaking, these models can reduce to the *R*-parity conserving minimal supersymmetric standard model (MSSM) with specific model dependent relationships among the Yukawa couplings. In addition to the seesaw constraints already provided by the neutrino physics (and the demand that these models reproduce all the low energy physics of the standard model), the Wilkinson Microwave Anisotropy Probe (WMAP) satellite observations [22] provide strong constraints on the available supersymmetric parameter space if the lightest supersymmetric particle (LSP) is assumed to compose the dark matter [23–25]. For a choice of constrained MSSM (CMSSM) parameters, the definite flavor structure of the AB model class results in specific predictions of lepton flavor violation and in particular the rate for $\mu \to e\gamma$. We determine how much of the currently viable CMSSM parameter space, as allowed by the WMAP observations, results in a $\mu \to e\gamma$ rate consistent with experimental limits for the AB model class.

We organize this paper as follows. In Sec. II we outline the essential details of the AB models, the supersymmetric parameter space, and the calculation for $\mu \to e\gamma$. We consider $\mu \to e\gamma$ since at the present time, with the current bound [26] of the branching ratio $\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$, this process gives the strongest constraints on lepton flavor violation in the class of models that we discuss. Furthermore, the MEG experiment at PSI [27] expects to improve on this bound with the expected sensitivity of $\mathrm{BR}(\mu \to e\gamma) \lesssim 5 \times 10^{-14}$. This experiment will provide stringent limits on models with charged lepton flavor violation. In Sec. III we display our numerical results on $\mu \to e\gamma$ together with the combined constraints from the WMAP satellite observations and direct search limits, and in Sec. IV we present our conclusions. The Appendix provides further calculational details.

II. THE AB MODEL DEFINITION

The AB model class is based on an SO(10) GUT with a $U(1) \times Z_2 \times Z_2$ flavor symmetry and uses a minimum set of Higgs fields to solve the doublet-triplet splitting problem [11–13]. The interesting feature of these models is the use of a lopsided texture. The approximate form of the charged lepton and the down quark mass matrix in these models is given by

$$\mathbf{Y}_{\mathrm{E}} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \sigma & 1 \end{pmatrix}, \quad \mathbf{Y}_{\mathrm{D}} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & \epsilon & 1 \end{pmatrix}, \tag{1}$$

where $\sigma \sim 1$ and $\epsilon \ll 1$. As pointed out by the authors of Ref. [11], this asymmetric structure naturally occurs within a minimal SU(5) GUT where the Yukawa interaction for the down quarks and leptons is of the form $\lambda_{ij}\overline{\mathbf{5}}_i \mathbf{10}_j \mathbf{5_H}$ ($\mathbf{5_H}$ denotes the Higgs scalars). In an SU(5) GUT, the left-handed leptons and the charge conjugate right-handed down quarks belong to the $\overline{\mathbf{5}}$ while the $\mathbf{10}$ contains the charge conjugate

*Electronic address: ejankows@phys.ualberta.ca
†Electronic address: dmaybury@phys.ualberta.ca

right-handed leptons and the left-handed down quarks. Therefore the lepton and down quark mass matrices are related to each other by a left-right transpose. Since SU(5) is a subgroup of SO(10), this feature is retained in an SO(10) GUT. This lopsided texture has the ability to explain why $|U_{\mu 3}| \gg |V_{cb}|$. Making use of this observation, the AB models contain the Dirac matrices $\mathbf{U},\mathbf{N},\mathbf{D},\mathbf{L}$ for the up-like quarks, Dirac neutrino interaction, down-like quarks, and the leptons, respectively [13],

$$\mathbf{U}=\begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \quad \mathbf{N}=\begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \tag{2}$$

$$\mathbf{D}=\begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D,$$

$$\mathbf{L}=\begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma+\epsilon & 1 \end{pmatrix} M_D, \tag{3}$$

where

$$\begin{aligned} &M_U \approx 113 \ \text{GeV}, \quad M_D \approx 1 \ \text{GeV}, \\ &\sigma = 1.78, \quad \epsilon = 0.145, \\ &\delta = 8.6\times 10^{-3}, \quad \delta' = 7.9\times 10^{-3}, \\ &\phi = 126°, \quad \eta = 8\times 10^{-6}. \end{aligned} \tag{4}$$

The given values of M_D and M_U best fit the low energy data with $\tan\beta \approx 5$. However, the mass scale itself is set only after electroweak symmetry breaking and it is therefore possible, with the use of $\tan\beta$, to extract dimensionless Yukawa matrices $\mathbf{Y}_{\mathrm{U}}$, $\mathbf{Y}_{\mathrm{N}}$, $\mathbf{Y}_{\mathrm{D}}$, and $\mathbf{Y}_{\mathrm{E}}$. It is advantageous to use dimensionless couplings since the renormalization group equations are initialized above the electroweak symmetry breaking scale. The corresponding dimensionless up- and down-like Yukawa matrices retain the form of Eqs. (2), and (3) but are scaled by two overall dimensionless factors: M_U' and M_D'. By varying the overall dimensionless scale factors, other values of $\tan\beta$ can be accommodated while retaining accurate fits to the low energy data after renormalization group running. Our code implements the one-loop beta functions [28–30] for the CMSSM with neutrino singlets and reproduces the results of Refs. [11–13]. Furthermore, we obtain accurate (within the stated errors in Ref. [31]) fits to the low energy data for $\tan\beta = 5$–50 which corresponds to $M_U' = 0.82$–0.85 and $M_D' = 0.016$–0.20.

The lopsided texture of the AB model class nicely fits the large atmospheric mixing angle; however, in order to obtain the large solar mixing angle a specific hierarchical form of the heavy Majorana singlet neutrino matrix needs to be chosen [12,13], namely,

$$\mathbf{M}_{\mathrm{N}}=\begin{pmatrix} b^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_{\mathrm{N}}, \tag{5}$$

where the parameters ϵ and η are as defined in Eq. (4). The parameters a and b are of order 1 and $\Lambda_{\mathrm{N}} \sim 2\times 10^{14}$ GeV. Since the Majorana singlet neutrino matrix is not related to the Dirac Yukawa structure, it is not surprising that this matrix should take on a form independent of the rest of the model. Once these choices have been made, the AB model class is highly predictive and accurately fits all the low energy standard model physics and the neutrino mixing observations.

It should be emphasized that all these relations are defined at the GUT scale and are therefore subject to renormalization group running. If we assume that the GUT symmetry breaks to the standard model gauge symmetries, $SU(3)\times SU(2)\times U(1)$, and that supersymmetry is broken supergravitionally through a hidden sector in a flavor independent manner, the AB model class will give well defined predictions for charged lepton flavor violation (LFV). There may also be contributions to the off-diagonal elements from renormalization group running between the GUT and gravity scales [32,33]. Since the particulars of GUT and supersymmetry breaking—as well as the possibility of new physics above the GUT scale—can have model dependent effects on the branching ratio for $\mu \to e\gamma$, we do not consider an interval of running between the GUT and gravity scales.

The specific model predictions for the Dirac Yukawa couplings and the form of the Majorana singlet neutrino matrix will feed into the soft supersymmetry breaking slepton mass terms through renormalization group running, generating off-diagonal elements that will contribute to flavor changing neutral currents [34]. The amount of flavor violation contained in the AB model class can be examined through the branching ratio of the process $\mu \to e\gamma$.

III. NUMERICAL RESULTS FOR $\mu \to e\gamma$

After GUT and supersymmetry breaking, the model class reduces to the CMSSM with heavy gauge singlet neutrinos to make use of the seesaw mechanism. It should be noted that given our assumptions about how the GUT and supersymmetry breaks, the CMSSM studies of Refs. [23–25] directly impact this model class. As discussed in the preceding section, the renormalization group running from the supersymmetry breaking scale to the weak scale alters the simple GUT relationship for the sfermion mass matrices. The diagonal part of the sfermion mass matrices is not strongly model dependent. The model dependence appears in the off-diagonal parts of the sfermion mass matrices that come from the particular textures of the model class—i.e., the mixings. Therefore, flavor changing neutral current processes are of primary interest, since they test the off-diagonal sfermion mass matrix structure. As discussed in the Introduction, $\mu \to e\gamma$ is the best constraining process for this model class. We note that the prediction for the anomalous magnetic moment of the muon in this model class is consistent with the

CMSSM analysis found in Ref. [23].

The leptonic part of the superpotential is

$$W=\epsilon_{\alpha\beta}H_{\rm d}^{\alpha}\mathbf{E}\mathbf{Y}_{\rm E}\mathbf{L}^{\beta}+\epsilon_{\alpha\beta}H_{\rm u}^{\alpha}\mathbf{N}\mathbf{Y}_{\rm N}\mathbf{L}^{\beta}+\frac{1}{2}\mathbf{N}\mathbf{M}_{\rm N}\mathbf{N}, \tag{6}$$

where $\mathbf{Y}_{\rm E}$, $\mathbf{Y}_{\rm N}$ are Yukawa matrices, and $\mathbf{M}_{\rm N}$ is the singlet Majorana neutrino mass matrix. The totally antisymmetric symbol is defined $\epsilon_{12}=+1$. We explain our notation in detail in the Appendix. On integrating out the heavy singlet neutrinos, Eq. (6) reduces to

$$W=\epsilon_{\alpha\beta}H_{\rm d}^{\alpha}\mathbf{E}\mathbf{Y}_{\rm E}\mathbf{L}^{\beta}-\frac{1}{2}\boldsymbol{\nu}^{\rm T}\mathbf{m}_{\nu}\boldsymbol{\nu}, \tag{7}$$

where

$$\mathbf{m}_{\nu}=\frac{v^2}{2}\mathbf{Y}_{\rm N}^{\rm T}\mathbf{M}_{\rm N}^{-1}\mathbf{Y}_{\rm N}\sin^2\beta \tag{8}$$

is the seesaw induced light neutrino mass matrix. The coefficients β and v are defined in terms of Higgs fields expectation values by

$$\frac{v^2}{2}=\langle H_{\rm d}^0\rangle^2+\langle H_{\rm u}^0\rangle^2=(174\ \ {\rm GeV})^2, \quad \tan\beta=\frac{\langle H_{\rm u}^0\rangle}{\langle H_{\rm d}^0\rangle}. \tag{9}$$

The neutrino mass matrix (8) is in general not diagonal and this is the source of lepton flavor violating interactions.

We assume that supersymmetry is broken softly in that breaking occurs through operators of mass dimension 2 and 3. The soft supersymmetry breaking Lagrangian relevant to LFV studies is

$$\begin{aligned}\mathcal{L}_{\rm breaking}=&-\delta_{\alpha\beta}\tilde{\mathbf{L}}^{\alpha\dagger}\mathbf{m}_{\tilde{\rm L}}^2\tilde{\mathbf{L}}^{\beta}-\tilde{\mathbf{E}}\mathbf{m}_{\tilde{\rm E}}^2\tilde{\mathbf{E}}^{\dagger}-\tilde{\mathbf{N}}\mathbf{m}_{\tilde{\rm N}}^2\tilde{\mathbf{N}}^{\dagger}\\&-m_{\rm H_d}^2\delta_{\alpha\beta}H_{\rm d}^{\alpha*}H_{\rm d}^{\beta}-m_{\rm H_u}^2\delta_{\alpha\beta}H_{\rm u}^{\alpha*}H_{\rm u}^{\beta}\\&+\left(-b\,\epsilon_{\alpha\beta}H_{\rm d}^{\alpha}H_{\rm u}^{\beta}-\frac{1}{2}\tilde{\mathbf{N}}\mathbf{B}_{\tilde{\rm N}}\tilde{\mathbf{N}}+{\rm c.c.}\right)\\&+(-\epsilon_{\alpha\beta}H_{\rm d}^{\alpha}\tilde{\mathbf{E}}\mathbf{A}_{\rm E}\tilde{\mathbf{L}}^{\beta}-\epsilon_{\alpha\beta}H_{\rm u}^{\alpha}\tilde{\mathbf{N}}\mathbf{A}_{\rm N}\tilde{\mathbf{L}}^{\beta}+{\rm c.c.})\\&+\left(-\frac{1}{2}M_1\tilde{B}\tilde{B}-\frac{1}{2}M_2\tilde{W}^a\tilde{W}^a+{\rm c.c.}\right)\end{aligned} \tag{10}$$

(see the Appendix for the notational details). The CMSSM assumes universal soft supersymmetry breaking parameters at the supersymmetry breaking scale, which we take to be of order the GUT scale, leading to the following GUT relations:

$$\mathbf{m}_{\tilde{\rm L}}^2=\mathbf{m}_{\tilde{\rm E}}^2=\mathbf{m}_{\tilde{\rm N}}^2=m_0^2\cdot\mathbf{I}, \tag{11}$$

$$m_{\rm H_d}^2=m_{\rm H_u}^2=m_0^2, \tag{12}$$

$$\mathbf{A}_{\rm E}=\mathbf{A}_{\rm N}=0, \tag{13}$$

$$M_1=M_2=m_{1/2}, \tag{14}$$

where m_0 and $m_{1/2}$ denote the universal scalar mass and the universal gaugino mass, respectively ($\mathbf{I}$ is the 3×3 unit matrix). We conservatively assume that the trilinear terms $\mathbf{A}_{\rm E}$ and $\mathbf{A}_{\rm N}$ vanish at the supersymmetry breaking scale.

We run the parameters of the CMSSM using the renormalization group equations [28–30] working in a basis where the Majorana neutrino singlet matrix is diagonal, integrating out each heavy neutrino singlet at its associated scale. After integrating down to the electroweak scale, we rotate the Yukawa couplings to the mass eigenbasis. In order to understand the origin of flavor violation in this model class, we first give a qualitative estimate. The leading log approximation of the off-diagonal slepton mass term is given by

$$(\Delta\mathbf{m}_{\tilde{\rm L}}^2)_{ij}\approx-\frac{3}{8\pi^2}m_0^2(\mathbf{Y_N}^{\dagger}\mathbf{Y_N})_{ij}\ln\left(\frac{M_{\rm GUT}}{\Lambda_{\rm N}}\right) \tag{15}$$

(assuming that the trilinears vanish at the GUT scale), and using this approximation together with mass insertion techniques [28,33], the branching ratio for $\mu\to e\gamma$ is

$$\begin{aligned}{\rm BR}(\mu\to e\gamma)&\sim\frac{\alpha^3}{G_{\rm F}^2}\frac{[(\mathbf{m}_{\tilde{\rm L}}^2)_{12}]^2}{m_{\rm s}^8}\tan^2\beta\\&\approx\frac{\alpha^3}{G_{\rm F}^2m_{\rm s}^8}\left|\frac{3}{8\pi^2}m_0^2\ln\frac{M_{\rm GUT}}{\Lambda_{\rm N}}\right|^2|(\mathbf{Y_N}^{\dagger}\mathbf{Y_N})_{12}|^2\tan^2\beta,\end{aligned} \tag{16}$$

where m_s is a typical sparticle mass. We see that since the flavor structure of the AB model class is specified so precisely, the branching ratio for $\mu\to e\gamma$ is well determined. In our calculation of the decay rate, we use the full one-loop expressions derived from the diagrams in Fig. 1 (see the Appendix for more details).

The WMAP satellite observations [22] strongly limit the available CMSSM parameter space if the LSP composes the dark matter [23–25]. We display our results over CMSSM parameter ranges determined by Refs. [23] and [24], which not only impose that the resulting model has LSP relic densities in the range determined by WMAP [22], but that they have spectra consistent with the CERN e^+e^- collider LEP direct search limits [31], as well as the rate for $b\to s\gamma$. Following these authors we ignore the focus point region in parameter space that occurs at very large m_0 and whose location depends on m_t and M_H in an extremely sensitive manner.

In Fig. 2, we show contours of the branching ratio $\mu\to e\gamma$ in the $m_{1/2}$-m_0 plane for a variety of $\tan\beta$ with the μ parameter both positive and negative. The parameters of the AB model class have been chosen such that all the low energy predictions fit the standard model data, and we have chosen $a=1$ and $b=2$ for the Majorana singlet neutrino mass matrix given in Eq. (5). As indicated in Ref. [13], there are a number of possible model choices for the Majorana singlet parameters a and b that are consistent with the LMA solution. However, we find that the rate for $\mu\to e\gamma$ is largely unaffected by the allowed range [13] for these parameters, $1.0\lesssim a\lesssim2.4$ and $1.8\lesssim b\lesssim5.2$. Figure 2 (a) demonstrates the lepton flavor bounds for $\tan\beta=5$ with $\mu>0$. The small

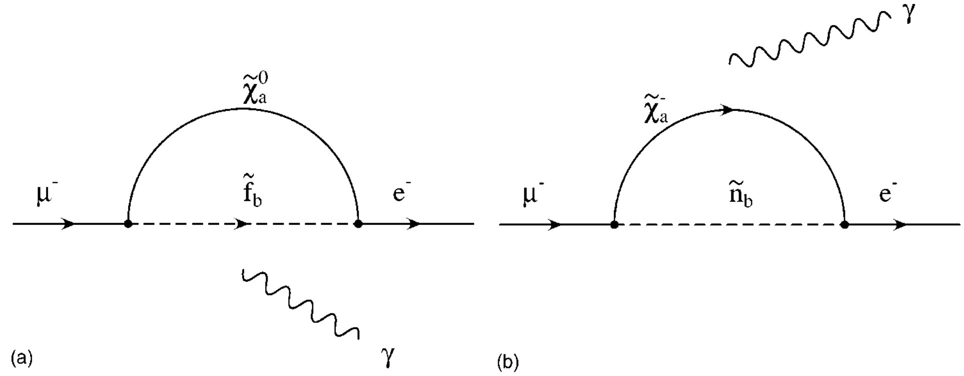

FIG. 1. Feynman diagrams contributing to $\mu \rightarrow e\gamma$.

linelike shaded area in the lower part of the panel is the allowed region from the combined WMAP and laboratory limits. The remaining panels show that the contours of constant branching ratio migrate to the right of the plots (i.e., to high values of $m_{1/2}$ and m_0) as $\tan\beta$ is increased. In each case we overlay the approximate WMAP and laboratory constraint bounds represented by a shaded region [23]. The choice for the sign of μ is indicated in each panel. As $\tan\beta$ is pushed up, larger portions of the parameter space become excluded. This is an expected feature since the branching ratio is proportional to $\tan^2\beta$. Notice that by $\tan\beta\sim 25$, $\mu>0$, the branching ratio allowed contours no longer have a significant overlap with the WMAP region. As a result, we find that the AB model class is consistent with the current experimental bound on $\mu\rightarrow e\gamma$ for low $\tan\beta$ (i.e., $\tan\beta \lesssim 20$) for $\mu>0$. For completeness, in panels (b) and (e), we show two cases where $\mu<0$. The branching ratio of $\mu\rightarrow e\gamma$ is largely insensitive to the sign of μ; however, the WMAP region is moderately affected [24]. A small part of the allowed WMAP region is currently permitted for larger $\tan\beta$ (i.e., ~ 35) as indicated in panel (e). The upcoming limits [27] that MEG will establish, $\mathrm{BR}(\mu\rightarrow e\gamma)\lesssim 5$

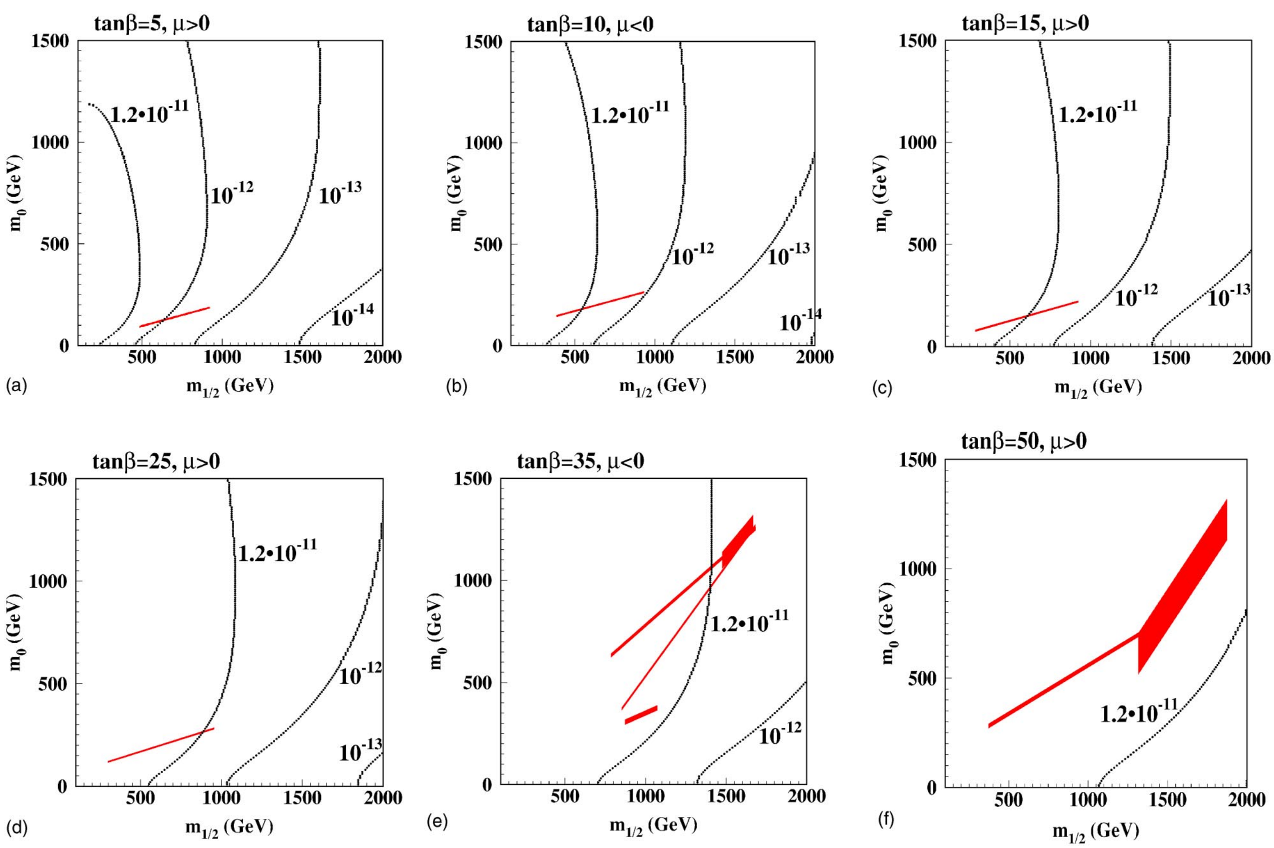

FIG. 2. Contour plots of $\mathrm{BR}(\mu\rightarrow e\gamma)$ in the $m_0 - m_{1/2}$ plane: Panels (a), (c), (d), and (f) show the contours of the branching ratio for $\tan\beta=5$, 15, 25, and 50 respectively, with $\mu>0$. Panels (b) and (e) show the contours with $\tan\beta=10$ and 35 respectively with $\mu<0$. In all cases the shaded region corresponds to the approximate combined WMAP and laboratory constraints.

$\times 10^{-14}$, will effectively rule out this model class if LFV is not seen. Interestingly, if LFV is seen at MEG, this model will suggest that $\tan\beta$ is low based on flavor bounds alone.

IV. CONCLUSIONS

The AB model class [11–13], based on a $U(1)\times Z_2\times Z_2$ flavor symmetry, is a highly successful and predictive GUT scenario. This model class has the ability to accommodate all the observed neutrino phenomena and reproduce the low energy physics of the standard model. If it is assumed that supersymmetry is broken via minimal supergravity (mSUGRA) and that the GUT breaks directly to the CMSSM, the AB model class is highly restrictive and hence allows for a precise determination for the rate of charged lepton flavor violation. In particular, we examined the process $\mu\to e\gamma$, since at the present time this flavor violating muon decay channel gives the strongest constraints on flavor changing neutral currents in the lepton sector.

As the WMAP satellite data [22] and laboratory direct searches [31] have already severely restricted the available CMSSM parameter space, the $\mu\to e\gamma$ flavor bounds allow a strong test of the AB model class. We find that given the current bounds [26] on $\mu\to e\gamma$, $\mathrm{BR}(\mu\to e\gamma)<1.2\times 10^{-11}$, the AB model class favors low to moderate $\tan\beta$ (i.e., $\lesssim 20$) with $\mu>0$; however, there is a small region that is not excluded for $\tan\beta\lesssim 35$ with the sign of μ negative. If MEG at PSI [27] does not detect a positive LFV signal, $\mathrm{BR}(\mu\to e\gamma)\lesssim 5\times 10^{-14}$, the AB model class will be effectively ruled out, given our conservative assumptions concerning GUT and supersymmetry breaking. It remains an open question as to whether or not other supersymmetry and/or GUT breaking schemes within the AB model class will be able to avoid these flavor violating bounds.

ACKNOWLEDGMENT

We would like to thank Bruce A. Campbell for helpful discussions and comments. D.M. acknowledges the support of the Natural Sciences and Engineering Research Council of Canada and E.J. the support of the Alberta Ingenuity Fund.

APPENDIX

In this section, we establish our notation and clarify some of the calculational details. We include the full one-loop amplitude for the rate $\mu\to e\gamma$ with generalized complex mixing matrices using the conventions that we detail below. It is highly probable that practitioners in the field are aware of the generalized rate expression, but to our knowledge it has not been explicitly stated in the literature. Formulas similar to those given here can be found in Ref. [28].

We express the supersymmetric Lagrangian using the two-component Weyl formalism. $\mathbf{L}^\alpha=(L_1^\alpha,L_2^\alpha,L_3^\alpha)^{\mathrm{T}}$ denotes a column vector in generation space containing the SU(2) doublet lepton chiral superfields; 1,2,3 are generation labels, and $\alpha=1,2$ are the SU(2) indices. $\mathbf{E}=(E_1,E_2,E_3)$ denotes a row vector in generation space containing SU(2) singlet charged lepton superfields. The gauge singlet neutrino chiral superfields are denoted by $\mathbf{N}=(N_1,N_2,N_3)$. Similarly, for the quark superfields: $\mathbf{Q}^\alpha=(Q_1^\alpha,Q_2^\alpha,Q_3^\alpha)^{\mathrm{T}}$ denotes the SU(2) doublet, $\mathbf{Q}^1=\mathbf{u}=(u_1,u_2,u_3)^{\mathrm{T}}$, $\mathbf{Q}^2=\mathbf{d}=(d_1,d_2,d_3)^{\mathrm{T}}$; and the SU(2) singlet quark superfields are $\mathbf{U}=(U_1,U_2,U_3)$, $\mathbf{D}=(D_1,D_2,D_3)$. H_{d}^α, H_{u}^α are the SU(2) Higgs doublet superfields of opposite hypercharge with the standard components: $H_{\mathrm{d}}^{\alpha=1}=H_{\mathrm{d}}^0$, $H_{\mathrm{d}}^{\alpha=2}=H_{\mathrm{d}}^-$, $H_{\mathrm{u}}^{\alpha=1}=H_{\mathrm{u}}^+$, $H_{\mathrm{u}}^{\alpha=2}=H_{\mathrm{u}}^0$. The corresponding scalar components of the superfields are written, respectively, as $\tilde{\mathbf{L}}^\alpha$, $\tilde{\mathbf{L}}^1=\tilde{\nu}$, $\tilde{\mathbf{L}}^2=\tilde{\mathbf{e}}$; $\tilde{\mathbf{E}}$; $\tilde{\mathbf{N}}$; $\tilde{\mathbf{Q}}^\alpha$, $\tilde{\mathbf{Q}}^1=\tilde{\mathbf{u}}$, $\tilde{\mathbf{Q}}^2=\tilde{\mathbf{d}}$; $\tilde{\mathbf{D}}$; $\tilde{\mathbf{U}}$ (all are vectors in generation space). The fermionic components of the Higgs superfield, the Higgsinos, are denoted as $\tilde{H}_{\mathrm{d}}^\alpha$, $\tilde{H}_{\mathrm{u}}^\alpha$. The superpotential W is given by

$$W=\epsilon_{\alpha\beta}H_{\mathrm{d}}^\alpha\mathbf{E}\mathbf{Y}_{\mathrm{E}}\mathbf{L}^\beta+\epsilon_{\alpha\beta}H_{\mathrm{u}}^\alpha\mathbf{N}\mathbf{Y}_{\mathrm{N}}\mathbf{L}^\beta+\frac{1}{2}\mathbf{N}\mathbf{M}_{\mathrm{N}}\mathbf{N}+\epsilon_{\alpha\beta}H_{\mathrm{d}}^\alpha\mathbf{D}\mathbf{Y}_{\mathrm{D}}\mathbf{Q}^\beta+\epsilon_{\alpha\beta}H_{\mathrm{u}}^\alpha\mathbf{U}\mathbf{Y}_{\mathrm{U}}\mathbf{Q}^\beta+\mu\epsilon_{\alpha\beta}H_{\mathrm{d}}^\alpha H_{\mathrm{u}}^\beta, \tag{A1}$$

where $\mathbf{Y}_{\mathrm{E}}$, $\mathbf{Y}_{\mathrm{N}}$, $\mathbf{Y}_{\mathrm{D}}$, $\mathbf{Y}_{\mathrm{U}}$ are Yukawa matrices, $\mathbf{M}_{\mathrm{N}}$ is the singlet Majorana neutrino mass matrix, μ is the Higgs parameter that breaks the U(1) Peccei-Quinn symmetry, and the totally antisymmetric symbol is defined $\epsilon_{12}=+1$. The soft supersymmetry breaking Lagrangian is

$$\begin{aligned}\mathcal{L}_{\mathrm{breaking}}=&-\delta_{\alpha\beta}\tilde{\mathbf{L}}^{\alpha\dagger}\mathbf{m}_{\tilde{\mathrm{L}}}^2\tilde{\mathbf{L}}^\beta-\tilde{\mathbf{E}}\mathbf{m}_{\tilde{\mathrm{E}}}^2\tilde{\mathbf{E}}^\dagger-\tilde{\mathbf{N}}\mathbf{m}_{\tilde{\mathrm{N}}}^2\tilde{\mathbf{N}}^\dagger-\delta_{\alpha\beta}\tilde{\mathbf{Q}}^{\alpha\dagger}\mathbf{m}_{\tilde{\mathrm{Q}}}^2\tilde{\mathbf{Q}}^\beta-\tilde{\mathbf{D}}\mathbf{m}_{\tilde{\mathrm{D}}}^2\tilde{\mathbf{D}}^\dagger-\tilde{\mathbf{U}}\mathbf{m}_{\tilde{\mathrm{U}}}^2\tilde{\mathbf{U}}^\dagger-m_{\mathrm{H_d}}^2\delta_{\alpha\beta}H_{\mathrm{d}}^{\alpha*}H_{\mathrm{d}}^\beta-m_{\mathrm{H_u}}^2\delta_{\alpha\beta}H_{\mathrm{u}}^{\alpha*}H_{\mathrm{u}}^\beta\\&+\left(-b\epsilon_{\alpha\beta}H_{\mathrm{d}}^\alpha H_{\mathrm{u}}^\beta-\frac{1}{2}\tilde{\mathbf{N}}\mathbf{B}_{\tilde{\mathrm{N}}}\tilde{\mathbf{N}}+\text{c.c.}\right)+(-\epsilon_{\alpha\beta}H_{\mathrm{d}}^\alpha\tilde{\mathbf{E}}\mathbf{A}_{\mathrm{E}}\tilde{\mathbf{L}}^\beta-\epsilon_{\alpha\beta}H_{\mathrm{u}}^\alpha\tilde{\mathbf{N}}\mathbf{A}_{\mathrm{N}}\tilde{\mathbf{L}}^\beta+\text{c.c.})+(-\epsilon_{\alpha\beta}H_{\mathrm{d}}^\alpha\tilde{\mathbf{D}}\mathbf{A}_{\mathrm{D}}\tilde{\mathbf{Q}}^\beta\\&-\epsilon_{\alpha\beta}H_{\mathrm{u}}^\alpha\tilde{\mathbf{U}}\mathbf{A}_{\mathrm{U}}\tilde{\mathbf{Q}}^\beta+\text{c.c.})+\left(-\frac{1}{2}M_1\tilde{B}\tilde{B}-\frac{1}{2}M_2\tilde{W}^a\tilde{W}^a-\frac{1}{2}M_3\tilde{G}^b\tilde{G}^b+\text{c.c.}\right),\end{aligned} \tag{A2}$$

where $\tilde{B}$ denotes the electroweak U(1) gaugino field; $\tilde{W}^a$, $a=1,2,3$, denote the electroweak SU(2) gaugino fields; $\tilde{G}^b$, $b=1,\dots,8$, denote the strong interaction, SU(3), gaugino fields; $\mathbf{m}_{\tilde{\mathrm{L}}}^2$, $\mathbf{m}_{\tilde{\mathrm{E}}}^2$, $\mathbf{m}_{\tilde{\mathrm{N}}}^2$, $\mathbf{m}_{\tilde{\mathrm{Q}}}^2$, $\mathbf{m}_{\tilde{\mathrm{D}}}^2$, $\mathbf{m}_{\tilde{\mathrm{U}}}^2$, $\mathbf{B}_\nu$, $\mathbf{A}_{\mathrm{E}}$, $\mathbf{A}_{\mathrm{N}}$, $\mathbf{A}_{\mathrm{D}}$, $\mathbf{A}_{\mathrm{U}}$, $m_{\mathrm{H_d}}^2$, $m_{\mathrm{H_u}}^2$, b, M_1, M_2, M_3 are the supersymmetry breaking parameters, and at the GUT scale:

$$\mathbf{m}_{\tilde{\mathrm{L}}}^2=\mathbf{m}_{\tilde{\mathrm{E}}}^2=\mathbf{m}_{\tilde{\mathrm{N}}}^2=\mathbf{m}_{\tilde{\mathrm{Q}}}^2=\mathbf{m}_{\tilde{\mathrm{D}}}^2=\mathbf{m}_{\tilde{\mathrm{U}}}^2=m_0^2\cdot\mathbf{I}, \tag{A3}$$

$$m_{H_d}^2 = m_{H_u}^2 = m_0^2, \tag{A4}$$

$$\mathbf{A}_E = \mathbf{A}_N = \mathbf{A}_D = \mathbf{A}_U = \mathbf{0}, \tag{A5}$$

$$M_1 = M_2 = M_3 = m_{1/2}, \tag{A6}$$

where m_0 and $m_{1/2}$ denote the universal scalar mass and the universal gaugino mass, respectively ($\mathbf{I}$ is the 3×3 unit matrix). After running the CMSSM renormalization group equations [28–30], we rotate all the Yukawa couplings to a diagonal basis, and in particular the lepton sector,

$$\mathbf{Y}_E \rightarrow \mathbf{U}_E^* \mathbf{Y}_E \mathbf{V}_E^\dagger = \text{diagonal}, \tag{A7}$$

$$\mathbf{m}_{\tilde{L}}^2 \rightarrow \mathbf{V}_E \mathbf{m}_{\tilde{L}}^2 \mathbf{V}_E^\dagger, \tag{A8}$$

$$\mathbf{m}_{\tilde{E}}^2 \rightarrow \mathbf{U}_E^* \mathbf{m}_{\tilde{E}}^2 \mathbf{U}_E^T, \tag{A9}$$

$$\mathbf{A}_E \rightarrow \mathbf{U}_E^* \mathbf{A}_E \mathbf{V}_E^\dagger. \tag{A10}$$

Not all of the bi-unitary rotation matrices can be absorbed away through the field redefinitions as the left-handed neutrinos become massive below the see-saw scale and after electroweak symmetry breaking.

Neutralinos

The neutralinos $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$ are mass eigenstates of the neutral gauginos $\tilde{B}$, $\tilde{W}^3$ and neutral Higgsinos $\tilde{H}_d^0$, $\tilde{H}_u^0$. The neutralino mass Lagrangian is given by

$$\mathcal{L} = -(\tilde{B}\ \tilde{W}^3\ \tilde{H}_d^0\ \tilde{H}_u^0)\mathbf{M}_{ne}\begin{pmatrix}\tilde{B}\\ \tilde{W}^3\\ \tilde{H}_d^0\\ \tilde{H}_u^0\end{pmatrix} + \text{c.c.}, \tag{A11}$$

where

$$\mathbf{M}_{ne} = \begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_W & m_Z\sin\beta\sin\theta_W \\ 0 & M_2 & m_Z\cos\beta\cos\theta_W & -m_Z\sin\beta\cos\theta_W \\ -m_Z\cos\beta\sin\theta_W & m_Z\cos\beta\cos\theta_W & 0 & -\mu \\ m_Z\sin\beta\sin\theta_W & -m_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix}. \tag{A12}$$

An orthonormal rotation leads to the mass eigenstates:

$$\begin{pmatrix}\tilde{\chi}_1^0\\ \tilde{\chi}_2^0\\ \tilde{\chi}_3^0\\ \tilde{\chi}_4^0\end{pmatrix} = \mathbf{O}_{ne}\begin{pmatrix}\tilde{B}\\ \tilde{W}^3\\ \tilde{H}_d^0\\ \tilde{H}_u^0\end{pmatrix}, \tag{A13}$$

where $\mathbf{O}_{ne}$ is a real, orthogonal matrix. The mass matrix (A12) can therefore be decomposed in terms of real mass eigenvalues, $M_{\tilde{\chi}_a^0}$, $a=1,2,3,4$,

$$\mathbf{M}_{ne} = \mathbf{O}_{ne}^T \text{diag}(M_{\tilde{\chi}_1^0}\ M_{\tilde{\chi}_2^0}\ M_{\tilde{\chi}_3^0}\ M_{\tilde{\chi}_4^0})\mathbf{O}_{ne}, \tag{A14}$$

and (A11) can be rewritten as

$$\mathcal{L} = -\frac{1}{2}\sum_{a=1}^{4} M_{\tilde{\chi}_a^0}\tilde{\chi}_a^0\tilde{\chi}_a^0. \tag{A15}$$

Charginos

The charginos are mass eigenstates of the charged SU(2) gauginos and charged Higgsinos,

$$\mathcal{L} = -(\tilde{W}^+\ \ \tilde{H}_u^+)\mathbf{M}_C\begin{pmatrix}\tilde{W}^-\\ \tilde{H}_d^-\end{pmatrix} + \text{c.c.}, \tag{A16}$$

where

$$\tilde{W}^\pm = \frac{\tilde{W}^1 \mp i\tilde{W}^2}{\sqrt{2}} \tag{A17}$$

and the mass matrix is

$$\mathbf{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{pmatrix} \tag{A18}$$

(m_W is the W-boson mass). The mass eigenstates are given by

$$\begin{pmatrix}\tilde{\chi}_1^-\\ \tilde{\chi}_2^-\end{pmatrix} = \mathbf{O}_L\begin{pmatrix}\tilde{W}^-\\ \tilde{H}_d^-\end{pmatrix}, \quad \begin{pmatrix}\tilde{\chi}_1^+\\ \tilde{\chi}_2^+\end{pmatrix} = \mathbf{O}_R\begin{pmatrix}\tilde{W}^+\\ \tilde{H}_u^+\end{pmatrix}, \tag{A19}$$

where $\mathbf{O}_R$ and $\mathbf{O}_L$ are real orthogonal matrices, and they can be chosen so that the mass eigenvalues $M_{\tilde{\chi}_1^-}$, $M_{\tilde{\chi}_2^-}$ are positive, and

$$\mathbf{M}_C = \mathbf{O}_R^T \text{diag}(M_{\tilde{\chi}_1^-}\ \ M_{\tilde{\chi}_2^-})\mathbf{O}_L. \tag{A20}$$

Equation (A16) can be written as

$$\mathcal{L} = -M_{\tilde{\chi}_1^-}\tilde{\chi}_1^+\tilde{\chi}_1^- - M_{\tilde{\chi}_2^-}\tilde{\chi}_2^+\tilde{\chi}_2^- + \text{c.c.} \tag{A21}$$

Sleptons

Masses of the charged sleptons are given by the Lagrangian

$$\mathcal{L}=-\tilde{\mathbf{e}}^{\dagger}\mathbf{m}_{\mathrm{LL}}^2\tilde{\mathbf{e}}-\tilde{\mathbf{e}}^{\dagger}\mathbf{m}_{\mathrm{RL}}^{2\dagger}\tilde{\mathbf{E}}^{\dagger}-\tilde{\mathbf{E}}\mathbf{m}_{\mathrm{RL}}^2\tilde{\mathbf{e}}-\tilde{\mathbf{E}}\mathbf{m}_{\mathrm{RR}}^2\tilde{\mathbf{E}}^{\dagger} \tag{A22}$$

with the mass matrices

$$\mathbf{m}_{\mathrm{LL}}^2=\mathbf{m}_{\mathrm{l}}^2+\mathbf{m}_{\tilde{\mathrm{L}}}^2+m_{\mathrm{Z}}^2\cos 2\beta\left(\sin^2\theta_{\mathrm{W}}-\frac{1}{2}\right)\cdot\mathbf{I}, \tag{A23}$$

$$\mathbf{m}_{\mathrm{RR}}^2=\mathbf{m}_{\mathrm{l}}^2+\mathbf{m}_{\tilde{\mathrm{E}}}^2-m_{\mathrm{Z}}^2\cos 2\beta\sin^2\theta_{\mathrm{W}}\cdot\mathbf{I}, \tag{A24}$$

$$\mathbf{m}_{\mathrm{RL}}^2=-\mu\mathbf{m}_{\mathrm{l}}\tan\beta+\frac{v\cos\beta}{\sqrt{2}}\mathbf{A}_{\mathrm{E}} \tag{A25}$$

where

$$\mathbf{m}_{\mathrm{l}}=\mathrm{diag}(m_{\mathrm{l}_1}\quad m_{\mathrm{l}_2}\quad m_{\mathrm{l}_3}), \tag{A26}$$

and m_{l_1}, m_{l_2}, m_{l_3} are electron, muon, and tau masses, respectively. The above Lagrangian written in terms of mass eigenstates $\tilde{f}_1,\ldots,\tilde{f}_6$ (six complex scalar fields) is

$$\mathcal{L}=-\sum_{b=1}^{6}m_{\tilde{\mathrm{f}}_b}^2\tilde{f}_b^*\tilde{f}_b \tag{A27}$$

with

$$\begin{pmatrix}\tilde{f}_1\\ \tilde{f}_2\\ \tilde{f}_3\\ \tilde{f}_4\\ \tilde{f}_5\\ \tilde{f}_6\end{pmatrix}=\mathbf{U}_{\tilde{\mathrm{f}}}\begin{pmatrix}\tilde{e}_1\\ \tilde{e}_2\\ \tilde{e}_3\\ \tilde{E}_1^*\\ \tilde{E}_2^*\\ \tilde{E}_3^*\end{pmatrix}, \tag{A28}$$

and $\mathbf{U}_{\tilde{\mathrm{f}}}$ is a complex unitary matrix defined by

$$\begin{pmatrix}\mathbf{m}_{\mathrm{LL}}^2 & \mathbf{m}_{\mathrm{RL}}^{2\dagger}\\ \mathbf{m}_{\mathrm{RL}}^2 & \mathbf{m}_{\mathrm{RR}}^2\end{pmatrix}=\mathbf{U}_{\tilde{\mathrm{f}}}^{\dagger}\mathrm{diag}(m_{\tilde{\mathrm{f}}_1}^2\quad m_{\tilde{\mathrm{f}}_2}^2\quad m_{\tilde{\mathrm{f}}_3}^2\quad m_{\tilde{\mathrm{f}}_4}^2\quad m_{\tilde{\mathrm{f}}_5}^2\quad m_{\tilde{\mathrm{f}}_6}^2)\mathbf{U}_{\tilde{\mathrm{f}}}. \tag{A29}$$

Similarly, the light sneutrinos (the heavy singlet sneutrinos are ignored since they have decoupled well above the weak scale)

$$\mathcal{L}=-\tilde{\boldsymbol{\nu}}^{\dagger}\mathbf{M}_{\tilde{\nu}}^2\tilde{\boldsymbol{\nu}}, \tag{A30}$$

where

$$\mathbf{M}_{\tilde{\nu}}^2=\mathbf{m}_{\tilde{\mathrm{L}}}^2+\frac{1}{2}m_{\mathrm{Z}}^2\cos 2\beta\cdot\mathbf{I}. \tag{A31}$$

The sneutrino mass Lagrangian written in terms of mass eigenstates $\tilde{n}_1$, $\tilde{n}_2$, $\tilde{n}_3$ (three complex scalar fields) reads

$$\mathcal{L}=-\sum_{b=1}^{3}m_{\tilde{\mathrm{n}}_b}^2\tilde{n}_b^*\tilde{n}_b \tag{A32}$$

with the mass eigenstates defined by

$$\begin{pmatrix}\tilde{n}_1\\ \tilde{n}_2\\ \tilde{n}_3\end{pmatrix}=\mathbf{U}_{\tilde{\mathrm{n}}}\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\\ \tilde{\nu}_3\end{pmatrix}, \tag{A33}$$

and $\mathbf{U}_{\tilde{\mathrm{n}}}$ is a complex unitary matrix satisfying

$$\mathbf{M}_{\tilde{\nu}}^2=\mathbf{U}_{\tilde{\mathrm{n}}}^{\dagger}\mathrm{diag}(m_{\tilde{\mathrm{n}}_1}^2\quad m_{\tilde{\mathrm{n}}_2}^2\quad m_{\tilde{\mathrm{n}}_3}^2)\mathbf{U}_{\tilde{\mathrm{n}}}. \tag{A34}$$

Lepton flavor violating interactions

The interactions leading to the lepton flavor violating process $l_j\to l_i+\gamma$ involve two effective Lagrangians: neutralino-lepton-slepton and chargino-lepton-sneutrino. Written in the mass eigenbasis they are

$$\mathcal{L}=\sum_{i=1}^{3}\sum_{a=1}^{4}\sum_{b=1}^{6}N_{iab}^{\mathrm{L}}\tilde{f}_bE_i\tilde{\chi}_a^0+N_{iab}^{\mathrm{R}*}\tilde{f}_b^*e_i\tilde{\chi}_a^0+\mathrm{c.c.} \tag{A35}$$

and

$$\mathcal{L}=\sum_{i=1}^{3}\sum_{a=1}^{2}\sum_{b=1}^{3}C_{iab}^{\mathrm{L}}\tilde{\nu}_bE_i\tilde{\chi}_a^-+C_{iab}^{\mathrm{R}*}\tilde{\nu}_b^*e_i\tilde{\chi}_a^++\mathrm{c.c.}, \tag{A36}$$

where

$$N_{iab}^{\mathrm{L}}=-\frac{g_2}{\sqrt{2}}\left(2\tan\theta_{\mathrm{W}}(\mathbf{U}_{\tilde{\mathrm{f}}})_{b(i+3)}^*(\mathbf{O}_{\mathrm{ne}})_{a1}+\frac{m_{\mathrm{l}_i}}{m_{\mathrm{W}}\cos\beta}(\mathbf{U}_{\tilde{\mathrm{f}}})_{bi}^*(\mathbf{O}_{\mathrm{ne}})_{a3}\right), \tag{A37}$$

$$N_{iab}^{\mathrm{R}}=\frac{g_2}{\sqrt{2}}\left(\tan\theta_{\mathrm{W}}(\mathbf{U}_{\tilde{\mathrm{f}}})_{bi}^*(\mathbf{O}_{\mathrm{ne}})_{a1}+(\mathbf{U}_{\tilde{\mathrm{f}}})_{bi}^*(\mathbf{O}_{\mathrm{ne}})_{a2}-\frac{m_{\mathrm{l}_i}}{m_{\mathrm{W}}\cos\beta}(\mathbf{U}_{\tilde{\mathrm{f}}})_{b(i+3)}^*(\mathbf{O}_{\mathrm{ne}})_{a3}\right), \tag{A38}$$

and

$$C_{iab}^{\mathrm{L}}=\frac{g_2m_{\mathrm{l}_i}}{\sqrt{2}m_{\mathrm{W}}\cos\beta}(\mathbf{O}_{\mathrm{L}})_{a2}(\mathbf{U}_{\tilde{\mathrm{n}}})_{bi}^*, \tag{A39}$$

$$C_{iab}^{\mathrm{R}}=-g_2(\mathbf{O}_{\mathrm{R}})_{a1}(\mathbf{U}_{\tilde{\mathrm{n}}})_{bi}^*. \tag{A40}$$

The on-shell amplitude for $l_j\to l_i+\gamma$ can be written in the general form

$$\mathcal{M}=e\epsilon_{\mu}^*\overline{l_i}(p-q)[(im_{\mathrm{l}_j}\sigma^{\mu\nu}q_{\nu}(A_{\mathrm{L}}\mathrm{L}+A_{\mathrm{R}}\mathrm{R})]l_j(p); \tag{A41}$$

here we have used Dirac spinors $l_i(p-q)$ and $l_j(p)$ for the charged leptons i and j with momenta $p-q$ and p, respectively; $\mathrm{L}=(1-\gamma^5)/2$ and $\mathrm{R}=(1+\gamma^5)/2$. Each of the dipole coefficients A_{L} and A_{R} have contributions from the neutralino-lepton-slepton and the chargino-lepton-sneutrino interaction, namely,

$$A_{\mathrm{L}}=A_{\mathrm{L}}^{(\mathrm{n})}+A_{\mathrm{L}}^{(\mathrm{c})}, \tag{A42}$$

$$A_{\mathrm{R}}=A_{\mathrm{R}}^{(\mathrm{n})}+A_{\mathrm{R}}^{(\mathrm{c})}, \tag{A43}$$

where $A_{\mathrm{L}}^{(\mathrm{n})}$, $A_{\mathrm{R}}^{(\mathrm{n})}$, $A_{\mathrm{L}}^{(\mathrm{c})}$, $A_{\mathrm{R}}^{(\mathrm{c})}$ can be evaluated from the Feynman diagrams in Fig. 1;

$$A_{\mathrm{L}}^{(\mathrm{n})}=\frac{1}{32\pi^2}\sum_{a=1}^{4}\sum_{b=1}^{6}\frac{1}{m_{\tilde{\mathrm{f}}_b}^2}\left[N_{iab}^{\mathrm{L}}N_{jab}^{\mathrm{L}*}J_1\left(\frac{M_{\tilde{\chi}_a^0}^2}{m_{\tilde{l}_b}^2}\right)+N_{iab}^{\mathrm{L}}N_{jab}^{\mathrm{R}*}\frac{|M_{\tilde{\chi}_a^0}|}{m_{l_j}}J_2\left(\frac{M_{\tilde{\chi}_a^0}^2}{m_{\tilde{l}_b}^2}\right)\right], \tag{A44}$$

$$A_{\mathrm{L}}^{(\mathrm{c})}=-\frac{1}{32\pi^2}\sum_{a=1}^{2}\sum_{b=1}^{3}\frac{1}{m_{\tilde{\nu}_b}^2}\left[C_{iab}^{\mathrm{L}}C_{jab}^{\mathrm{L}*}J_3\left(\frac{M_{\tilde{\chi}_a^-}^2}{m_{\tilde{\nu}_b}^2}\right)+C_{iab}^{\mathrm{L}}C_{jab}^{\mathrm{R}*}\frac{M_{\tilde{\chi}_a^-}}{m_{l_j}}J_4\left(\frac{M_{\tilde{\chi}_a^-}^2}{m_{\tilde{\nu}_b}^2}\right)\right], \tag{A45}$$

$$A_{\mathrm{R}}^{(\mathrm{n})}=A_{\mathrm{L}}^{(\mathrm{n})}\big|_{L\leftrightarrow R}, \tag{A46}$$

$$A_{\mathrm{R}}^{(\mathrm{c})}=A_{\mathrm{L}}^{(\mathrm{c})}\big|_{L\leftrightarrow R}. \tag{A47}$$

The functions $J_1(x)$, $J_2(x)$, $J_3(x)$, $J_4(x)$ are defined as

$$J_1(x)=\frac{1-6x+3x^2+2x^3-6x^2\ln x}{6(1-x)^4}, \tag{A48}$$

$$J_2(x)=\frac{1-x^2+2x\ln x}{(1-x)^3}, \tag{A49}$$

$$J_3(x)=\frac{2+3x-6x^2+x^3+6x\ln x}{6(1-x)^4}, \tag{A50}$$

$$J_4(x)=\frac{-3+4x-x^2+2\ln x}{(1-x)^3}. \tag{A51}$$

Finally, the decay rate for $l_j^-\rightarrow l_i^-+\gamma$ is given by

$$\Gamma(l_j^-\rightarrow l_i^-+\gamma)=\frac{e^2}{16\pi}m_{l_j}^5(|A_{\mathrm{L}}|^2+|A_{\mathrm{R}}|^2), \tag{A52}$$

and $i=1$, $j=2$ for $\mu\rightarrow e+\gamma$.

[1] Homestake Collaboration, R. Davis *et al.*, Rev. Mod. Phys. **75**, 985 (2003).

[2] SAGE Collaboration, J.N. Abdurashitov *et al.*, Phys. Lett. B **328**, 234 (1994); Phys. Rev. Lett. **83**, 4686 (1999); J. Exp. Theor. Phys. **95**, 181 (2002).

[3] GALLEX Collaboration, P. Anselmann *et al.*, Phys. Lett. B **285**, 237 (1992); **357**, 390 (1995); **361**, 235(E) (1996); **477**, 127 (1999); **490**, 16 (2000).

[4] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5651 (2001); **86**, 5656 (2001).

[5] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001); **89**, 011301 (2002); **89**, 011302 (2002); **92**, 181301 (2004).

[6] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); **82**, 2644 (1999); **85**, 3999 (2000).

[7] KAMLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).

[8] K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).

[9] For reviews see P. Ramond, *Journeys Beyond The Standard Model* (Perseus, Boulder, CO, 1999); M. Fukugita and T. Yanagida, *Physics of Neutrinos* (Springer-Verlag, Berlin, 2003).

[10] K.S. Babu and S.M. Barr, Phys. Lett. B **381**, 202 (1996).

[11] Carl H. Albright and S.M. Barr, Phys. Rev. D **62**, 093008 (2000).

[12] C.H. Albright and S.M. Barr, Phys. Rev. D **64**, 073010 (2001).

[13] C.H. Albright and S. Geer, Phys. Lett. B **532**, 311 (2002).

[14] K.S. Babu, J.C. Pati, and F. Wilczek, Nucl. Phys. **B566**, 33 (2000).

[15] Z. Berezhiani and A. Rossi, Nucl. Phys. **B594**, 113 (2001).

[16] T. Blazek, S. Raby, and K. Tobe, Phys. Rev. D **62**, 055001 (2000).

[17] W. Buchmuller and D. Wyler, Phys. Lett. B **521**, 291 (2001).

[18] M.-C. Chen and K.T. Mahanthappa, Phys. Rev. D **65**, 053010 (2002).

[19] R. Kitano and Y. Mimura, Phys. Rev. D **63**, 016008 (2001).

[20] N. Maekawa, Prog. Theor. Phys. **106**, 401 (2001).

[21] G.G. Ross and L. Velasco-Sevilla, Nucl. Phys. **B653**, 3 (2003).

[22] WMAP Collaboration, C.L. Bennett *et al.*, Astrophys. J., Suppl. **148**, 1 (2003); WMAP Collaboration, D.N. Spergel *et al.*, *ibid.* **148**, 175 (2003).

[23] J.R. Ellis, K.A. Olive, Y. Santoso, and V.C. Spanos, Phys. Lett. B **565**, 176 (2003).

[24] M. Battaglia, A. De Roeck, J.R. Ellis, F. Gianotti, K.A. Olive, and L. Pape, Eur. Phys. J. C **33**, 273 (2004).

[25] J.R. Ellis, K.A. Olive, Y. Santoso, and V.C. Spanos, Phys. Rev. D **69**, 095004 (2004); H. Baer and C. Balazs, J. Cosmol. Astropart. Phys. **05**, 006 (2003); H. Baer, C. Balazs, A. Belyaev, T. Krupovnickas, and X. Tata, J. High Energy Phys. **06**, 054 (2003); A.B. Lahanas and D.V. Nanopoulos, Phys. Lett. B **568**, 055 (2003); U. Chattopadhyay, A. Corsetti, and P. Nath,

Phys. Rev. D **68**, 035005 (2003); R. Arnowitt, B. Dutta, and B. Hu, hep-ph/0310103.

[26] MEGA Collaboration, M.L. Brooks *et al.*, Phys. Rev. Lett. **83**, 1521 (1999).

[27] T. Mori, Nucl. Phys. B (Proc. Suppl.) **111**, 194 (2002); the MEG website: http://meg.web.psi.ch/

[28] J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996).

[29] S.P. Martin and M.T. Vaughn, Phys. Rev. D **50**, 2282 (1994).

[30] N.K. Falck, Z. Phys. C **30**, 247 (1986).

[31] Particle Data Group, Phys. Rev. D **66**, (2002).

[32] F. Barbieri, L. Hall, and A. Strumia, Nucl. Phys. **B445**, 219 (1995).

[33] J. Hisano and D. Nomura, Phys. Rev. D **59**, 116005 (1999).

[34] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).